Mechanism Design for Stochastic Dynamic Parking Resource Allocation

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1. Introduction

In practice, although parking slot occupation information can be sensed by a centralized operator in real time, some information essential for centralized slot assignment, such as drivers' parking duration and final destinations, is usually not centrally available but is only individually known. Inevitably, this provides incentives for the strategic behaviors of drivers (a driver may maximize her utility by misreporting private information), possibly leading to the suboptimal system performance of a centralized parking management platform. Resolving this problem calls for a mechanism design to align drivers' selfish behavior with the objective of the centralized operator to achieve an optimal allocation of parking resources. In the literature, for simplicity, many studies investigating mechanism designs for parking management (e.g., Chen et al., 2015; Zou et al., 2015) assume that all drivers are willing to participate in a parking slot assignment and that a driver will not incur a negative utility if she participates and is eventually not assigned a slot. However, in reality, because of the cost of opportunities, a negative utility will indeed emerge if a driver participates and is eventually not assigned a slot.

The contributions of this paper are threefold. First, we propose a mechanism design to manage parking space allocation in a dynamic and stochastic environment. To guarantee its effectiveness, the proposed mechanism design is based on an ADP approach. Second, we relax the assumption that a driver's utility remains nonnegative if she chooses a parking system operated under a mechanism design and is not assigned a parking slot. In this context, a rational driver will maximize her expected utility to decide whether or not to choose the parking system. Third, we show that the resulting equilibrium under the proposed mechanism design may not always be the approximate system optimum entry condition because of the equilibrium's non-uniqueness. We provide an integrated mechanism design as a corrective.

2. Methodology

In each period, the actions of the central decision maker (CDM) are divided into two stages. In the first stage, the CDM decides which drivers are allowed into the system to maximize the overall expected discounted benefits of the system. A driver who enters the system and is not assigned a slot may incur a large penalty, as the waiting time at stages one and two constitutes an opportunity cost and she may thus incur a delay in arriving at her destination. Therefore, the CDM may block the entry of some drivers since the available parking spaces are limited. In the second stage, the CDM determines the optimal parking space allocation.

In period t, the CDM's first-stage optimal decision-making problem can be formulated as follows:

$$V_t(\mathcal{W}_t) = \max_{\mathcal{G}_t} \mathbb{E}_{\mathcal{R}_t} \Psi_t(\mathcal{W}_t, \mathcal{G}_t, \mathcal{R}_t) \quad (1)$$

Given $\mathcal{G}_t$, the CDM's decision-making problem in the second stage can be formulated as follows:

$$\Psi_t(\mathcal{W}_t, \mathcal{G}_t, \mathcal{R}_t) = \max_{\mathcal{X}_t} C_t(\mathcal{W}_t, \mathcal{R}_t, \mathcal{G}_t, \mathcal{X}_t) + \gamma V_{t+1}(\mathcal{W}_{t+1})$$

(2)

$s.t.$

$$\sum_{n \in N} x_{in}^t \leq 1 \qquad \forall i \in \mathcal{I}_t'$$

$$\sum_{i \in \mathcal{I}_t} x_{in}^t \leq 1 \qquad \forall n \in \Omega_t$$

$$\xi_i - x_{in}^t g_{od}^{mt} \geq 0 \qquad \forall i \in I_{od}^{mt}, n \in \Omega_t, (o,d,m) \in \mathcal{B}$$

$$\tau_{t+1}^n = max(\tau_t^n - 1, 0) \qquad \forall n \in \mathcal{N} \backslash \Omega_t$$

$$\tau_{t+1}^n = \sum_{(o,d,m) \in \mathcal{B}} \sum_{i \in I_{od}^{mt}} m x_{in}^t \qquad \forall n \in \Omega_t$$

$$x_{in}^t \in \{0,1\} \qquad \forall i \in \mathcal{I}_t',\ n \in \Omega_t$$

Denote by $Q_t(\mathcal{W}_t, \mathcal{G}_t)$ the expected discounted value of being in state $\mathcal{W}_t$ and taking action $\mathcal{G}_t$, i.e., $Q_t(\mathcal{W}_t, \mathcal{G}_t) \coloneqq \mathbb{E}_{\mathcal{R}_t} \Psi_t(\mathcal{W}_t, \mathcal{G}_t, \mathcal{R}_t)$. To tackle the computational complexity of solving the above stochastic programming, we develop an ADP approach wherein we adopt the value function approximation $\bar{Q}_t(\mathcal{W}_t, \mathcal{G}_t)$ to replace $Q_t(\mathcal{W}_t, \mathcal{G}_t)$ in Bellman's equation and step forward in time to make decisions, i.e.,

$$\max_{\mathcal{X}_t \in \bar{\mathcal{X}}_t, \mathcal{G}_{t+1}} C_t(\mathcal{W}_t, \mathcal{R}_t, \mathcal{G}_t, \mathcal{X}_t) + \gamma \bar{Q}_{t+1}(\mathcal{W}_{t+1}, \mathcal{G}_{t+1}) \ (3)$$

In this process, we gradually reach more accurate value function approximations and better policies by iteratively updating the value function approximations.

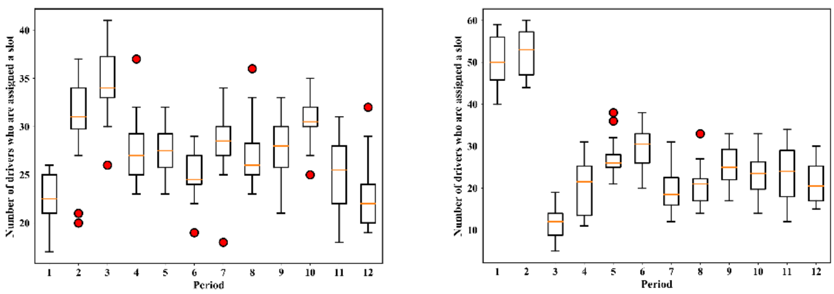
To facilitate the implementation of an optimal allocation of parking resources, we then propose an online mechanism design whereby drivers' rational and strategic behaviors comply with CDM's approximate optimal decisions, $\mathcal{G}_t^*$ and $\mathcal{X}_t^*$. Specifically, the parking fees drivers need to pay are calculated through $p_i(\hat{\theta}_t) =$

$-\left(\sum_{j\neq i,j\in\widehat{\mathcal{D}}_t}\hat{r}_j'(X_{1t})+\gamma\bar{V}_{t+1}\left(\mathcal{W}_{t+1}(\mathcal{W}_t,\hat{\theta}_t^i,\hat{\theta}_t^{-i},X_{1t})\right)\right)+\left(\sum_{j\neq i,j\in\widehat{\mathcal{D}}_t}\hat{r}_j'(X_{2t})+\gamma\bar{V}_{t+1}\left(\mathcal{W}_{t+1}(\mathcal{W}_t,\hat{\theta}_t^{-i},X_{2t})\right)\right)$, where $X_{1t}\in arg\max_X\sum_{j\in\widehat{\mathcal{D}}_t}\hat{r}_j'(X)+\gamma\bar{V}_{t+1}(\mathcal{W}_{t+1}(\mathcal{W}_t,\hat{\theta}_t^i,\hat{\theta}_t^{-i},X))$ and $X_{2t}\in arg\max_X\sum_{j\neq i,j\in\widehat{\mathcal{D}}_t}\hat{r}_j'(X)+\gamma\bar{V}_{t+1}(\mathcal{W}_{t+1}(\mathcal{W}_t,\hat{\theta}_t^{-i},X))$.

We then prove that, in stage one, drivers' choices of whether or not to enter the managed system following the operator's recommendation satisfy Bayesian-Nash equilibrium, and in stage two, that truthful reporting is a dominant strategy for all drivers under any circumstance.

3. Results

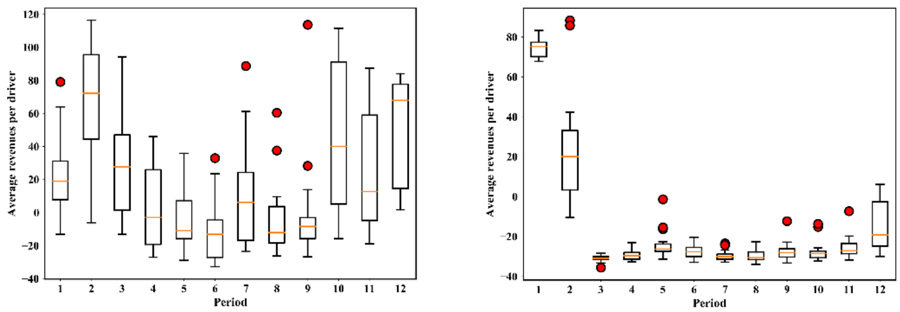
Figure 1 compares the numbers of drivers choosing to enter the publically-owned parking system and are finally assigned parking slots under OM^*(online mechanism design) and MP (myopic policy). We observe that the CDM allocates fewer parking slots to drivers in the first two periods under OM^*, while fewer parking slots are allocated to drivers in the latter periods under MP. This makes intuitive sense because the CDM only considers the current system revenues and thus tends to utilize all available parking spaces in each period. In contrary, the OM^* takes into account future system revenue and thus may reserve some parking slots for latter parking demands such that the system revenues of all periods can be maximized.



(a) OM^* (b) MP

Figure 1. The number of drivers who are assigned parking slots

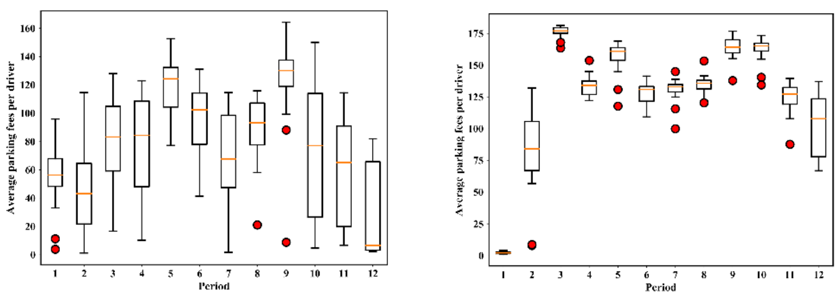
Figure 2 shows the average realized revenues of drivers choosing to enter the publically-owned parking system. The average realized revenues under MP remains relatively stable over many period, which is attributed to the excessive utilization of parking slots in earlier periods. That is, if there are few parking slots available, then the CDM's decision space is rather limited, and hence the average realized driver revenues are of less fluctuation.



(a) OM^* (b) MP

Figure 2. Average realized revenues of drivers entering the system

Figure 3 compares the average parking fee each driver needs to pay if assigned a parking slot. In general, the parking fees under OM^* yields a more substantial fluctuation than those under MP, which is attributed to the fact that the CDM's decision space in the latter periods is rather limited under MP. Furthermore, the average parking fee under OM^* is lower than that under MP except the first period. The parking fee a driver pays reflects the impact of her entry on the system, and this impact is rather modest if there are more parking slots available.



(a) OM^* (b) MP

Figure 3. Average parking fees per driver

4. Conclusion

In this paper, we first investigate the stochastic dynamic parking resource allocation problem for a publicly owned parking system. To address the curses of dimensionality and the complexity of state transition probability, we adopt an ADP framework to derive an approximate optimal solution by value iterations. To cope with strategic and rational behaviors of drivers, based on the approximate value function, we further propose a two-stage online mechanism design to induce the approximate optimal allocation of parking resources.

In future research, to guarantee the computational tractability, we will investigate how to design value function approximations such that the entry condition derived by the ADP method exactly satisfies the BNE condition. Furthermore, we will explore designing a charging scheme in the online mechanism design that yields a unique BNE condition.

References

Anderson, S. P., De Palma, A. (2004). The economics of pricing parking. Journal of Urban Economics, 55(1), 1-20.

Balinski, M. L., Gomory, R. E. (1964). A primal method for the assignment and transportation problems. Management Science, 10(3), 578-593.

Bertsekas, D. P. (1988). The auction algorithm: A distributed relaxation method for the assignment problem. Annals of operations research, 14(1), 105-123.

Birge, J. R., Louveaux, F. (2011). Introduction to stochastic programming. Springer Science & Business Media.

Blum, A., Kumar, V., Rudra, A., Wu, F. (2004). Online learning in online auctions. Theoretical Computer Science, 324(2-3), 137-146.

Fosgerau, M., Palma, A. D. (2013). The dynamics of urban traffic congestion and the price of parking. Journal of Public Economics, 105(4), 106-115.

Gallien, J. (2006). Dynamic mechanism design for online commerce. Operations Research, 54(2), 291-310.

He, F., Yin, Y., Chen, Z., Zhou, J. (2015). Pricing of parking games with atomic players. Transportation Research Part B: Methodological, 73, 1-12.

Heydenreich, B., Müller, R., Uetz, M. (2010). Mechanism design for decentralized online machine scheduling. Operations research, 58(2), 445-457.

Huh, W. T., Liu, N., Truong, V. A. (2013). Multiresource allocation scheduling in dynamic environments. Manufacturing & Service Operations Management, 15(2), 280-291.

Kang, K. , Shanthikumar, J. G. , Altinkemer, K. . (2016). Postponable acceptance and assignment: a stochastic dynamic programming approach. Manufacturing & Service Operations Management, 18.

Lavi, R., Nisan, N. (2004). Competitive analysis of incentive compatible on-line auctions. Theoretical Computer Science, 310(1-3), 159-180.

Lei, C., Ouyang, Y. (2017). Dynamic pricing and reservation for intelligent urban parking management. Transportation Research Part C Emerging Technologies, 77, 226-244.

Liu, W., Geroliminis, N. (2016). Modeling the morning commute for urban networks with cruising-for-parking: an mfd approach. Transportation Research Part B Methodological, 93, 470-494.

Long, E. F., Nohdurft, E., Spinler, S. (2018). Spatial Resource Allocation for Emerging Epidemics: A Comparison of Greedy, Myopic, and Dynamic Policies. Manufacturing & Service Operations Management, 20(2), 181-198.

Maskin, E., Riley, J. (1984). Optimal auctions with risk averse buyers. Econometrica: Journal of the Econometric Society, 1473-1518.

Mashayekhy, L., Nejad, M. M., Grosu, D., Vasilakos, A. V. (2016). An online mechanism for resource allocation and pricing in clouds. IEEE transactions on computers, 65(4), 1172-1184.

Nasrollahzadeh, A. A., Khademi, A., Mayorga, M. E. (2018). Real-Time Ambulance Dispatching and Relocation. Manufacturing & Service Operations Management, 20(3), 467-480.

Parkes, D. C., Singh, S. (2003). An MDP-based approach to online mechanism design. International Conference on Neural Information Processing Systems (pp.791-798). MIT Press.

Psaraftis, H. N. . (1995). Dynamic vehicle routing: status and prospects. Annals of Operations Research, 61(1), 143-164.

Parkes, D. C., Yanovsky, D., & Singh, S. P. (2005). Approximately efficient online mechanism design. In Advances in Neural Information Processing Systems (pp. 1049-1056).

Powell, W. B. . (1996). A stochastic formulation of the dynamic assignment problem, with an application to truckload motor carriers. Transportation Science, 30(3), 195-219.

Powell, W. B. (2007). Approximate Dynamic Programming: Solving the curses of dimensionality (Vol. 703). John Wiley & Sons.

Qian, Z., Xiao, F., Zhang, H. M. (2011). The economics of parking provision for the morning commute. Transportation Research Part A Policy & Practice, 45(9), 861-879.

Qian, Z., Rajagopal, R. (2014). Optimal dynamic parking pricing for morning commute considering expected cruising time. Transportation Research Part C, 48, 468-490.

Richard Arnott, Andre de Palma, Robin Lindsey. (1991). A temporal and spatial equilibrium analysis of commuter parking. Journal of Public Economics, 45(3), 301-335.

Secomandi, N. . (2001). A rollout policy for the vehicle routing problem with stochastic demands. Operations Research, 49(5), 796-802.

Shapley, L. S. 1962. Complements and substitutes in the optimal assignment problem. Naval Res. Logist. Quart. 9 45–48.

Spivey, M. Z., Powell, W. B. (2004). The dynamic assignment problem. Transportation Science, 38(4), 399-419.

Ströhle, P., Gerding, E. H., de Weerdt, M. M., Stein, S., Robu, V. (2014, May). Online mechanism design for scheduling non-preemptive jobs under uncertain supply and demand. In Proceedings of the 2014 international conference on Autonomous agents and multi-agent systems (pp. 437-444). International Foundation for Autonomous Agents and Multiagent Systems.

Ulmer, M. W. , Goodson, J. C. , Mattfeld, D. C. , Hennig, M. . (2018). Offline-online approximate dynamic programming for dynamic vehicle routing with stochastic requests. Transportation Science.

Vulcano, G., Van Ryzin, G., Maglaras, C. (2002). Optimal dynamic auctions for revenue management. Management Science, 48(11), 1388-1407.

Zhang, X., Yang, Z., Zhou, Z., Cai, H., Chen, L., Li, X. (2014). Free market of crowdsourcing: Incentive mechanism design for mobile sensing. IEEE transactions on parallel and distributed systems, 25(12), 3190-3200.

Zou, B., Kafle, N., Wolfson, O., Lin, J. (2015). A mechanism design based approach to solving parking slot assignment in the information era. Transportation Research Part B, 81, 631-653.